Conductance quantization: A laboratory experiment in a senior-level nanoscale science and technology course

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Conductance quantization: A laboratory experiment in a senior-level nanoscale science and technology course

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We describe a simple, inexpensive, and robust undergraduate lab experiment that demonstrates the emergence of quantized conductance as a macroscopic gold wire is broken and unbroken. The experiment utilizes a mechanically controlled break junction and demonstrates how conductance quantization can be used to understand the importance of quantum mechanics at the nanoscale. Such an experiment can be integrated into the curriculum of a course on nanoscale science or contemporary physics at the junior and senior levels.
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I. INTRODUCTION

The past two decades have witnessed an enormous surge of interest in nanotechnology and nanoscience. This interest is fueled by predictions that nanotechnology will have a significant and broad impact on many aspects of the future, including technology,[1] food,[2] medicine,[3,4] and sustainable energy.[5] Many universities in the United States and around the world began to establish programs teaching nanotechnology in order to produce the necessary nanoscale-skilled workforce[6–8] and to inform the public about nanotechnology's potential benefits and environmental risks.[9,10] Nanoscale science and technology programs have even been utilized to sustain low-enrollment physics programs and to reform the Science, Technology, Engineering, and Mathematics (STEM) focus.[11] However, it is necessary to devise additional experiments and to develop curricula that will motivate the field properly and provide undergraduate students with a good appreciation and basic understanding of the nanoscale.[12,13] Several core concepts have been identified as fundamental to student understanding of phenomena at the nanoscale.[14] Two such concepts are the importance of quantum mechanics and the understanding of the sizes and scales at which interesting phenomena occur. Quantum mechanics shows that when matter is confined at the atomic scale, it can have quantitatively and qualitatively different properties than the macroscopic scale.[15] One consequence of this confinement and the particle-wave duality is the quantization of electrical conductance, where the classical electron transport properties and the well-established Ohm's Law cease to apply.[16]

We develop a simple, inexpensive, and robust laboratory experiment on conductance quantization that can be used as an example of the emergence of new behavior at the nanoscale. Our setup employs the Mechanically Controlled Break Junction (MCBJ) technique to form an atomic-scale constriction in a bulk gold wire.[17,18] Starting with a wire with a weak point, the ductile nature of gold allows the constriction, or weak area, to shrink while stretching the wire until there are only a few atoms left at the constriction. A single-atom chain then forms just before the wire breaks. Because the nature of gold allows the wire to reconnect and break again easily and repeatedly, this process can be repeated as many times as desired using the same wire. While conductance quantization experiments have been utilized and integrated into course curricula[16,19] and even in a public exhibit,[20] our approach is unique in that it does not need any advanced lithography yet gives excellent reproducibility and control of the breaking and reconnecting of the wire. It also costs much less to make the samples[20] and uses a simpler measurement setup, as compared to the setup in Ref. 20. The experiment has two nice pedagogical features. First, it helps students understand that confinement at the nanoscale leads to observable quantum-mechanical effects. And second, the different transport and scattering regimes can serve as natural "milestones" in appreciating the size scales involved in reducing a conductor's dimensions from the macro- to the nanoscale.

This experiment was developed for a senior-level course on nanoscale science and technology offered in the physics department. Nearly half of the students in the course are engineering majors. The meetings for this class are split evenly between classroom learning and hands-on laboratory experiments. Topics for direct experience through experimentation include lithography, microscopy, and characterization of nanoscale features and materials. This experiment is performed in a single two-hour class. In the classroom students use the textbook *Nanophysics and Nanotechnology*,[21] and get an introduction to basic quantum mechanics as well as many other aspects of nanotechnology. Their study also includes a self-directed investigation of an individually chosen aspect of nanoscale science. Most of the students already have experience with LABVIEW programming and are familiar with both data acquisition and analysis; this allows the conductance experiment to focus on the importance of wave properties of matter at the nanoscale, as well as the different behavior present at each size scale.

II. THEORY

A. Classical model for charge transport in a wire

The simple (classical) Drude model[22] assumes that conduction electrons in a metal move freely and randomly in all directions within the metal, similar to the particles in an ideal gas. Such motion is depicted by the solid (blue) arrows in Fig. 1(a). The "thermal" speed of the electrons depends on the temperature T and is given by[22] $\langle v\rangle = \sqrt{8k_BT/\pi m}$, where m is the electron mass, $\langle v\rangle$ the average speed, and k_B the Boltzmann constant. The average distance that an electron travels before it scatters is known as the mean-free-path l, and the net velocity of an electron in the absence of external forces is zero because the electrons move randomly in all directions.

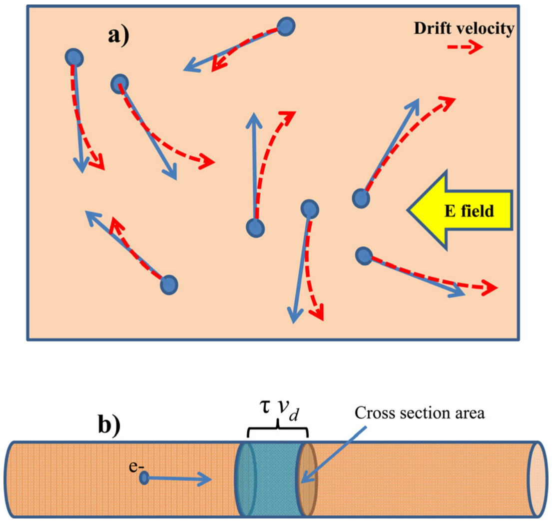

Fig. 1. (Color online) The flow of free electrons in metals gives the electric current. (a) In the absence of electric fields, electrons move randomly in all directions (solid blue lines) and have a net velocity of zero. When a field is applied, the electrons accelerate in a direction opposite the field (dotted red curves) and there will be a net drift velocity that is responsible for the electric current. (b) The current is the total charge that passes a cross-sectional area A per unit second, or equivalently the charge density times the volume of the charge that crosses plane A every second.

When a potential difference V is applied across a wire, it produces an electric field E and a force F acting on the electrons in a direction opposite the field. Thus, an electron will accelerate between collisions according to $\vec{F} = m\vec{a} = -e\vec{E}$ and its speed after time t from being scattered is given by $\vec{v}_2 = \vec{v}_0 + e\vec{E}t/m$, where $\vec{v}_0$ is the electron speed immediately after being scattered. When averaged over the time between collisions, one obtains the drift velocity,

$$v_d = \frac{eE\tau}{m}, \tag{1}$$

where τ is the average time between collisions. The effect of the electric field on electron trajectories is depicted by the dotted (red) arrows shown in Fig. 1(a). The curvature in the arrows is not to scale because the thermal speed of the electrons is typically about 10 orders of magnitude higher than the net drift velocity.[22]

The electric current in a wire of cross-sectional area A is the total charge passing a given point each second. If N is the number density of free electrons in the metal, then the current is given by (see Fig. 1),

$$I = \frac{\Delta Q}{\Delta t} = eNAv_d. \tag{2}$$

Substituting from Eq. (1) leads to the usual form of Ohm's law,[23]

$$J = \frac{e^2 NE\tau}{m} = \sigma E, \tag{3}$$

where J is the current density and $\sigma = e^2 N\tau/m$ is the conductivity, an intrinsic property of the material that does not depend on the geometry. The conductance $G = I/V$ of a wire of length L is then given by $G = \sigma A/L$.

This simple (classical) model works reasonably well and needs only two quantum-mechanical correction—replacing v_d by the Fermi velocity v_F and treating the electron as a wave instead of a hard sphere—to yield correct values of σ for macroscopic metals.[22] But this treatment fails when the sample size is small (comparable to the electron mean-free-path), when the conductance becomes independent of the sample length and varies in discrete steps rather than being continuous.

B. Transport in a wire with a constriction: The importance of size and scale

If we take a macroscopic wire and make a constriction of width w and length L, then the proper understanding and calculation of the conductance depends on the relative sizes of w and L compared to the mean-free-path and the de Broglie wavelength at the Fermi surface (λ_F) of the electrons in the wire. Specifically, there are three limits that produce different conduction properties across the constriction: $w,\ L \gg l$, $L < l$, and $w \approx \lambda_F$. These three limits are discussed below.

1. *The classical limit*

Figure 2(a) shows a pictorial representation of a wire with a constriction such that $w,\ L \gg l$, the classical limit. In this case, an electron traveling through the constriction will scatter many times before it reaches the end of the constriction. Because the wire is a metal there will be no charge accumulation anywhere within the constriction so the Laplace equation $\nabla^2 V(x, y, z) = 0$ applies. In this case, the conductance is given by[16]

$$G = w\sigma, \tag{4}$$

showing that the conductance is a smooth function of the radius of the constriction in the classical limit, which applies to macroscopic conductors.

2. *The semi-classical limit*

As shown in Fig. 2(b), when the constriction length is much less than l, the transport of electrons will occur without any scattering and the electrons will accelerate with no momentum loss in the constriction. Such a situation is referred to as ballistic transport. To model the behavior of electrons in this limit requires a mixture of concepts from quantum and classical mechanics and is therefore called the semi-classical limit.[24] The conductance in this limit is known as the Sharvin conductance and is given by[16,25] $G = (2e^2/h)(k_F w/4)^2$, where h is Planck's constant and k_F is the wave vector at the Fermi energy. The conductance of the constriction in this limit is independent of the material conductivity and increases quadratically with its width.

3. *The quantum limit*

As the constriction radius shrinks further and gets down to the atomic scale, it will be comparable to the de Broglie wavelength of the electrons at the Fermi surface $w \approx \lambda_F$. At this point, a full quantum-mechanical treatment is necessary to understand the system behavior. The hallmark of this transport limit is that the conductance is quantized. If we model the constriction to be very long in the x-direction (the direction of net electron motion) and to have a small width in the radial direction ($w \ll L$), then this radial confinement will cause the radial motion to be quantized, allowing only a

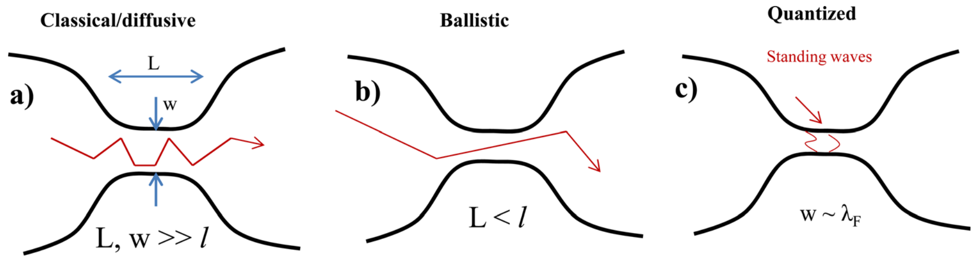

Fig. 2. (Color online) The relative length L and width w of the constriction to the mean-free-path and Fermi wavelength determine its conductance properties. (a) In the diffusive regime, electrons scatter many times while in the constriction, so the classical theory describes the transport properties well. (b) The ballistic regime is when the mean-free-path is longer than the constriction and no scattering takes place in the constriction area. (c) As the constriction width becomes comparable to the Fermi wavelength, the wave nature of the electrons dominates the transport and only electrons with given wavelengths (or channels) are allowed to move across the constriction.

finite number of wavelengths or "conduction channels" in this direction [Fig. 2(c)]. The x-motion will still be continuous, but the number of conduction channels in the constriction is limited, similar to a one-dimensional infinite square well of width w, where $\lambda_n = h/p_n = 2w/n$, where p_n and λ_n are, respectively, the momentum and the de Broglie wavelength of an electron in quantized level n.

If we consider all states below the Fermi energy to be occupied and all states above it to be empty, then the shortest de Broglie wavelength is fixed at the Fermi wavelength $\lambda_F = h/\sqrt{2m\epsilon_F}$, where ϵ_F is the Fermi energy. This means the number of conduction channels n depends directly on the width ($n = 2w/\lambda_F$), and as the width of the constriction becomes smaller the number of allowed channels decreases in integer steps, due to the quantization of the allowed wavelengths. When the width of the constriction is reduced to one gold atom ($\sim$0.25 nm), the width is equal to half the Fermi wavelength and only one conduction channel is allowed.[16] When a voltage is applied across the constriction, the magnitude of the current for a single conduction channel k is given by

$$I_k = 2e\int_0^\infty v_k(\epsilon)[\rho_{kL}(\epsilon) - \rho_{kR}(\epsilon)]\,d\epsilon, \tag{5}$$

where v_k is the Fermi velocity of electrons in channel k, the factor of 2 is due to spin degeneracy, ϵ is the energy, L and R refer to the left and right sides of the constriction, and ρ is the one-dimensional density of states: $\rho = \sqrt{m/2h^2\epsilon} = 1/hv$ for $\epsilon < \epsilon_f$, and $\rho = 0$ for $\epsilon > \epsilon_f$.[19] The above integrand is zero except in the range $\epsilon_F - eV/2$ to $\epsilon_F + eV/2$ (or just 0 to eV), because this is where the density of states differs on the left and right. The net current is therefore

$$I_k = 2e\int_0^{eV} v_k\left(\frac{1}{hv_k} - 0\right)d\epsilon = 2\frac{e^2}{h}V, \tag{6}$$

which gives the (quantized) conductance per channel as $G_k = 2e^2/h$. This conductance value is twice the fundamental unit of conductance (due to spin degeneracy), and is independent of material properties and geometry. For an integer number of channels n, the conductance is

$$G_n = 2\frac{e^2}{h}n. \tag{7}$$

Thus, as the constriction narrows the number of available channels decreases in integer steps, giving rise to the quantized conductance effect seen in this experiment.

III. EXPERIMENTAL SETUP AND MEASUREMENTS

Our MCBJ setup uses a spring-steel sheet as a bending beam and a micrometer to stretch a 99.99%-pure, 3.5″-long and 75 μm-wide gold wire with atomic displacement accuracy. The SM-25 Vernier Micrometer has a resolution of about 1 μm and is rotated manually by attaching it to a plastic disc of radius 5″. The 1095 Blue Tempered Spring-Steel sheet is a little over 3″ long, 0.5″ wide, and 0.008″ in thickness. The barrel of the micrometer passes through a hole in an aluminum housing block and is secured by a set screw. When fully retracted, the micrometer head is flush with the aluminum block. Two stops are placed 3″ apart, centered on the hole for the micrometer head. These conductive stops are electrically insulated from the main aluminum block by a length of plastic tubing, and are positioned so that there is 4 mm of distance between the fully retracted micrometer head and the plane of the stops. The sample is placed in this space and the micrometer head is advanced to make contact with the sample. With the barrel of the micrometer secured in place, the tip can be extended and retracted by rotating the thimble. As the tip extends it presses into the middle of the spring-steel sheet, bending the spring steel outwards against the two stops and producing the desired bending motion. If the sample is particularly long, the ends of the spring-steel sheet may contact the aluminum block as it bends. This is prevented by cutting two clearance notches on either side of the block. Figure 3 shows pictures of the setup used in this experiment.

Because the spring-steel sheet is conductive we cover it with a thin insulating layer of Krylon spray paint and then attach the gold wire using two droplets of Double/Bubble insulating epoxy as shown in Fig. 3(c). After the epoxy hardens, we use a sharp blade to cut a shallow notch in the gold wire. The blade is also used to cut a groove in the epoxy if the two droplets merge together. Figure 3(d) shows a scanning electron microscope image of the partly cut wire and the two epoxy drops. We also used cigarette paper instead of the spray-on insulation to electrically isolate the conductive bending beam from the gold wire. Both approaches worked well.

When turning the plastic disk and micrometer, the wire stretches extremely slow with a reduction factor f given by $f = 3ys/u^2$, where y is the distance between the two epoxy drops, s is the thickness of the spring-steel sheet and insulating film, and u is the separation between the two stopping edges. We estimate $f \sim 2 \times 10^{-5}$ (corresponding to a mechanical reduction of 50,000), which, when multiplied by the

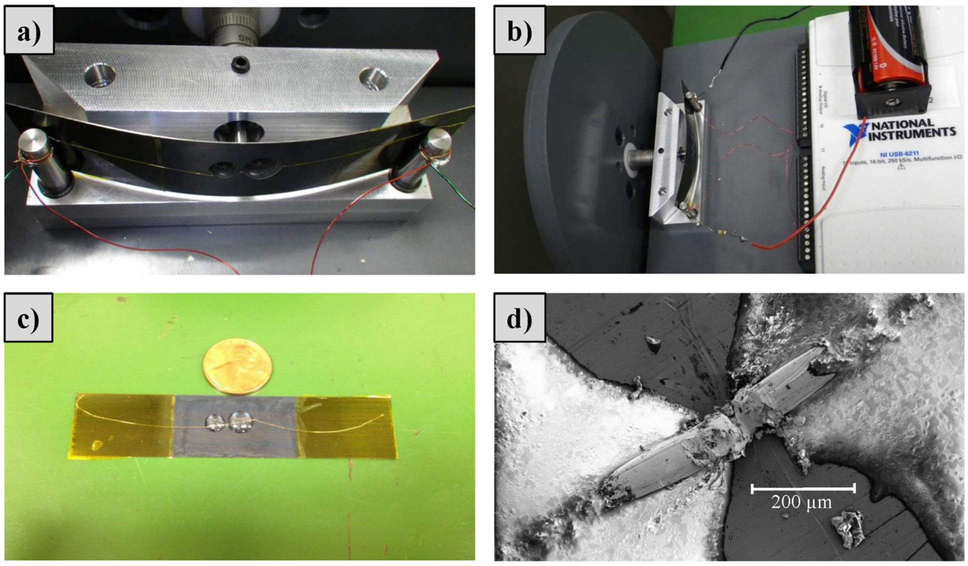

Fig. 3. (Color online) Pictures of the MCBJ setup. (a) The MCBJ assembly showing the pin of the micrometer, the bending beam, the stops, and the wire. No solder is needed to connect the ends of the gold wire to the stops. (b) The experimental setup showing the plastic disk used to rotate the micrometer, the battery and wires, and the bending beam. (c) A gold wire mounted on a sheet of spring steel with a quarter-dollar coin next to it for visual comparison. The two epoxy drops are seen in the middle. (d) Scanning electron microscope image of the wire and the two drops. The wire is partially cut in the middle to create a week point.

micrometer resolution of 1 μm, gives atomic-scale motion. The huge reduction in the bending beam is the key to achieve atomic-scale motion and to eliminate the effect of external vibrations on the experiment.[16]

The current through the constriction is produced by connecting the wire in series to an external 100-kΩ resistor and a 1.5-V battery. As the wire is pulled, the voltage across it is measured repeatedly at a high rate (10,000 samples per second) using a National Instruments data acquisition (DAQ) unit and a simple LABVIEW program. The circuit diagram and the LABVIEW program used to collect the data are shown in Fig. 4.

Previous experiments have used tapping on a table to connect and disconnect two (separate but touching) gold wires, among other approaches,[19,26,27] and they display clear quantized conductance steps. However, our MCBJ setup offers better stability as well as control over the breaking and reconnecting of the gold wire. A conductance step may last for tens to hundreds of milliseconds at a time in this MCBJ setup, rather than microseconds as in other experiments.[19] Furthermore, our resistance measurement setup is much simpler and more direct, making our approach better suited to undergraduate labs.

Another recent experiment uses MCBJs to demonstrate conductance quantization in a public exhibit.[20] However, this experiment requires deep-UV lithography or electron-beam lithography to make the break junctions. Such a fabrication requirement makes this approach difficult to adopt in most physics labs that do not have extensive nano-fabrication capabilities. Another pedagogical advantage of our approach is that by not using advanced lithography, students are not distracted from appreciating the vastly different length scales[14,28] that are spanned by the shrinking constriction radius. The entire experiment occurs right before the students' eyes. Our break junctions are made from macroscopic wires and the setup is simple, inexpensive (each sample costs around $1.75 and can be used repeatedly), and

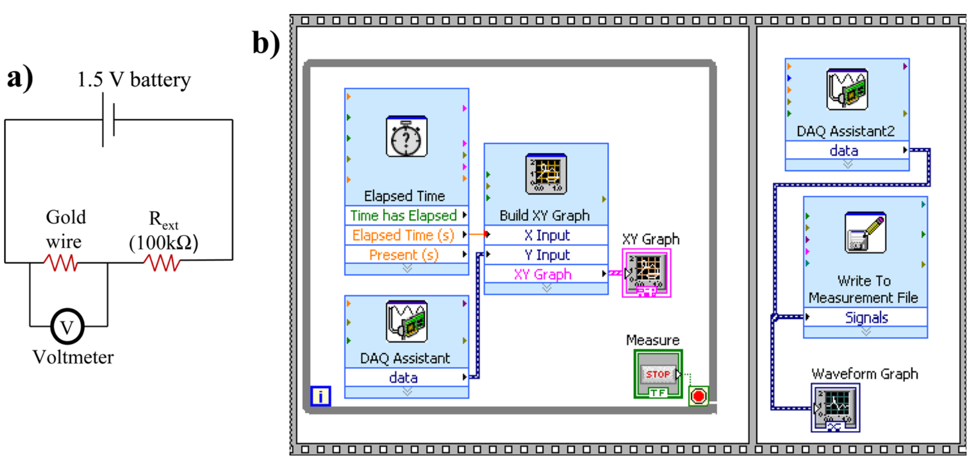

Fig. 4. (Color online) (a) A simple electrical circuit is used to measure the conductance of the gold wire and (b) a very basic LABVIEW program monitors and records the data.

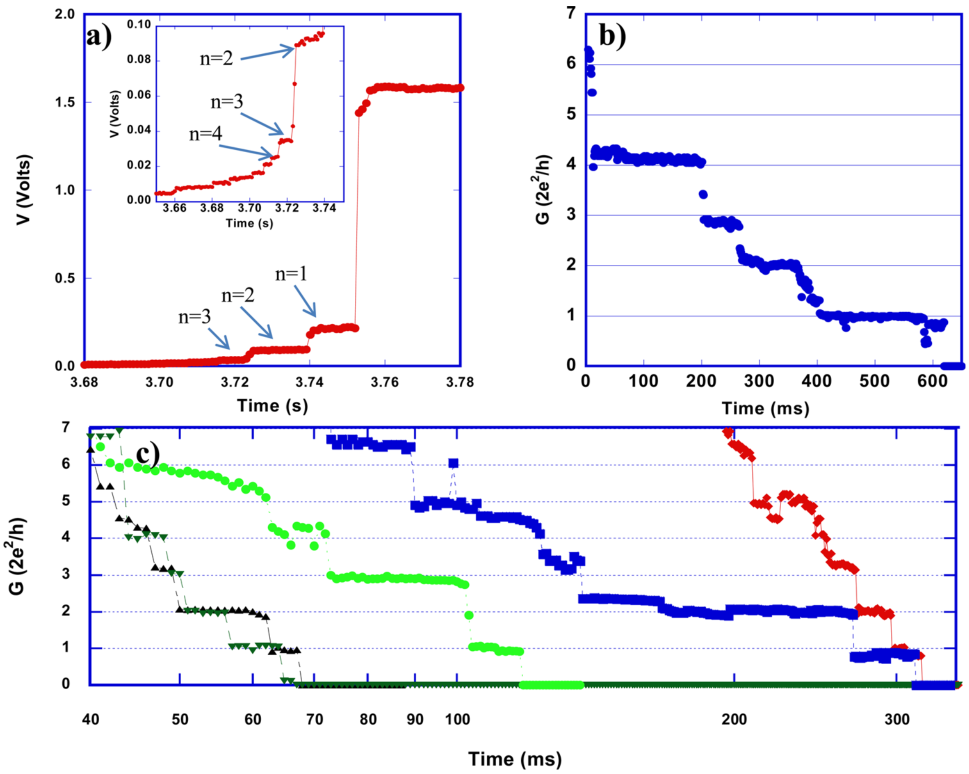

Fig. 5. (Color online) Quantized conductance data. (a) The voltage across the constriction varies in a stepwise manner due to the quantized resistance of the constriction. Inset shows the same graph for a smaller voltage range. The voltage step size gets smaller with increasing n. (b) Conductance in fundamental conductance units is shown versus time; G is quantized and clear steps are observed at integer values of n. (c) Several data sets in one graph collected from a single wire. Each of the runs displays quantized conductance. Time is displayed on a logarithmic scale.

accessible to advanced undergraduates in most science and engineering programs.

IV. RESULTS AND DISCUSSION

Starting with the unbroken wire, the plastic disc is rotated slowly, turning the attached micrometer. As the constriction stretches, its diameter shrinks and the voltage across the wire rises continuously because the wire resistance increases with decreasing diameter. When the constriction diameter becomes comparable to the de Broglie wavelength of the electrons (the Fermi wavelength), the voltage displays discrete steps rather than a smooth increase. Figure 5(a) shows the voltage variation with time as the wire is being stretched until it breaks. Because the wire is connected in series to the external 100-kΩ resistor, the voltage across the constriction is $V_w = IR_w = R_w V_B/(R_w + R_{ext})$, giving a conductance of

$$G = \frac{V_B - V_w}{V_w R_{ext}}, \tag{8}$$

where V_B is the battery voltage, R_{ext} is the external resistor, and R_w is the resistance of the wire (i.e., the constriction). Figure 5(b) shows a plot of conductance versus time in units of $2e^2/h$. It is clear that G decreases when stretching the wire and makes quantized jumps that coincide with integer values of n.

Students were comfortable performing all steps of the experiment, and the entire experiment can be completed within a two-hour lab session. Figure 5(c) shows multiple conductance measurement runs taken on the same wire that broke and reconnected several times. Conductance quantization and the reproducibility of the results are clearly visible.

V. CONCLUSIONS

We have built a simple and robust experimental setup to demonstrate and measure the quantized conductance in an atomic-scale constriction in a macroscopic gold wire. This experiment can be repeated as many times as desired and can be taught as a laboratory experiment in a junior- or senior-level course on nanoscience and nanotechnology, or in other advanced laboratories.

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Brachistrochrone Demonstration

One of the staples of the graduate-level advanced mechanics course is the use of the calculus of variations to solve the problem of finding the curve that brings a body, in a friction-free environment, from one level to another in the least time. This problem was solved by Newton in one day in 1696. The required curve, a segment of a cycloid, is called a *Brachistrochrone*. This example, in the Greenslade Collection, has a straight-line track and two cycloidal tracks. The reason for two identical tracks is to demonstrate the paradoxical property of the cycloid curve that two bodies released from different elevations reach the bottom at the same time. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College)